

# Dihyperon in Chiral Colour Dielectric model

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## Abstract

The mass of dihyperon with spin, parity  $J^\pi = 0^+$  and isospin  $I = 0$  is calculated in the framework of Chiral colour dielectric model. The wave function of the dihyperon is expressed as a product of two colour-singlet baryon clusters. Thus the quark wave functions within the cluster are antisymmetric. Appropriate operators are then used to antisymmetrize inter-cluster quark wave functions. The radial part of the quark wavefunctions are obtained by solving the the quark and dielectric field equations of motion obtained in the Colour dielectric model. The mass of the dihyperon is computed by including the colour magnetic energy as well as the energy due to meson interaction. The recoil correction to the dihyperon mass is incorporated by Peierls-Yoccoz

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technique. We find that the mass of the dihyperon is smaller than the  $\Lambda - \Lambda$  threshold by over 100 MeV. The implications of our results on the present day relativistic heavy ion experiments is discussed.

The possibility of the existence of a stable six quark dibaryon composed of two up (u), two down (d) and two strange (s) quarks confined in a single hadronic bag and having spin, parity  $J^\pi = 0^+$  and isospin  $I = 0$  was first proposed by Jaffe in 1977 [1]. This object is a singlet of colour, flavour and spin and thus results in a maximally attractive colour magnetic interaction between the quarks. Jaffe's calculation predicted the mass of dihyperon ( $H_1$ ) to be around 2150 MeV which is about 80 MeV less than the  $\Lambda - \Lambda$  threshold. Such a state would then be stable against strong decays into three-quark baryons and can decay only by weak interaction into a pair of baryons. Jaffe's calculation was performed in the MIT bag model. Later, this calculation was refined by including center-of-mass correction[2], SU(3)-flavour symmetry breaking[3], surface energy term in the bag model[4], coupling of pseudo-scalar meson octate [5] etc. Calculations have also been performed in non-relativistic potential model[6, 7, 8] and Skyrmion model[9]. Production cross sections of dihyperons in various experiments have also been estimated[10]. Most of these calculations predict the mass of  $H_1$  very close to  $\Lambda - \Lambda$  threshold. Some of these calculations predict that  $H_1$  is stable against strong decays with mass below the  $\Lambda - \Lambda$  threshold where as the others predict an unstable  $H_1$ . If the dihyperon mass is below the  $\Lambda - \Lambda$  threshold, one expects that, unlike the deuteron, the dihyperon would be a state of six-quarks bound in a single bag and not a two-baryon state bound by meson exchange interaction. The reason for this is that in the deuteron the pion exchange interaction provides the bulk of the binding force where as in the dihyperon case, one-pion exchange is not possible in  $\Lambda - \Lambda$  channel and therefore the meson exchange contribution to the binding is expected to be small. Therefore the experimental determination of the dihyperon mass is expected to impose a strong constraint on the quark models used in hadron spectroscopy.

With these considerations, the experimental as well as theoretical investigations of dihyperons in particular and dibaryons in general is of great interest. As is well known, the QCD is the theory of strong interactions and in such a theory, six-quark colour-neutral objects are expected to exist. Whether these are stable against strong

decays depends on the details. Already a number of QCD-inspired models have been employed to investigate the properties of baryons, the three-quark colour-neutral objects and generally these models are quite successful. The calculations of the properties of dihyperons and other dibaryons in these models and comparison of these with the experimental results is needed as these models are likely to yield different results in the dibaryon sector. The experimental observation ( or otherwise ) of the dihyperon will then be able to indicate which of these models are better.

The dihyperon, if stable, is likely to be produced in relativistic heavy ion collisions due to the abundant strangeness production in the hot and dense hadronic matter formed in the collision. For example, the calculations using a cascade code like ARC [16] find that in a collision of Au ions with similarly heavy target nuclei, more than 20 hyperons are expected to be produced in central collisions at the AGS energies. This implies that there is a large probability of  $\Lambda - \Lambda$  coalescence leading to the formation of a dihyperon. Further more, the formation of quark gluon plasma with large baryon density and its subsequent decay is also expected to enhance the strangeness production in the fragmentation region. This would lead to an enhancement in the formation of dihyperons in the relativistic heavy ion collisions. So far, evidence for the existence of dihyperon or otherwise is rather scanty and inconclusive. An isolated H candidate has been reported in bubble chamber experiments [17]. Three H particle candidates have been observed in three different emulsion experiments [18]. In another experiment of heavy ion collision of Au + Pt, E886 collaboration has reported a null result for the search of H particle [19]. On the other hand in a more recent heavy ion experiment [20] a number of dihyperons seem to have been detected. This experiment seems to give the dihyperon mass of about 2180 MeV and the lifetime of about  $3.3 \times 10^{-10}$  sec. Of course, it must be noted that this result is not yet conclusive enough.

In the present work the mass of dihyperon has been calculated in the framework of chiral colour dielectric (CCD) model. The CCD model has been used earlier in baryon spectroscopy [21] and for the investigation of static properties of nucleons in

nuclear medium[22]. These calculations have shown that the model is able to explain the static properties of light baryons very well. Furthermore, when applied to the quark matter calculation, the model yields an equation of state which is quite similar to the one obtained from lattice calculations for zero baryon chemical potential [23]. The CCD model differs from the bag model in several aspects. First of all, in the CCD model, the confinement of quarks and gluons is achieved dynamically through the colour-dielectric field. In the bag model this is done by hand. Also the quark masses used in the CCD model are different from those used in the bag model. In the bag model,  $u$  and  $d$  masses are taken to be zero. The CCD model requires that these masses are nonzero. It has been found [21] that to fit baryon masses, the required  $u$  and  $d$  masses are  $\sim 100MeV$ . It might be noted that similar masses are used in relativistic quark models [24]. Thus, the values of quark masses in the CCD model are closer to the constituent quark masses. The main difference between the present calculation and most of the earlier dihyperon calculations is the inclusion of the pseudoscalar meson coupling to the dihyperon state ( however see [5] and [8] which include meson self energy ). The meson self-energy corrections are expected to shift the masses of dibaryons by a few hundreds of MeV, just like the shifts produced in the baryon masses. Therefore one needs to include these self energy contributions in a more realistic calculation. Further more, the explicit breaking of SU(3)-flavour symmetry, which arises from the difference between strange and  $u$  (  $d$  ) quark masses is included in our calculation.

The methodology adopted in the present work is similar to the one followed in the baryon spectroscopy calculations[21], or for that matter the one followed in the cloudy bag model calculations[25]. Thus we first compute the quark wavefunctions in the mean field approximation by solving coupled quark and dielectric field equations obtained in the CCD model. These equations are

$$(\vec{\alpha} \cdot \vec{p} - m - \frac{m_0}{\chi(r)})\Psi(r) = E\Psi(r) \quad (1)$$

$$\chi''(r) + \frac{2}{r}\chi'(r) - \frac{\partial V(\chi)}{\partial \chi} + \frac{m_0}{\sigma_v^2 \chi^2(r)} < \bar{\Psi}(r)\Psi(r) > = 0. \quad (2)$$

Here  $\Psi(r)$  is the four-component Dirac spinor,  $\chi(r)$  is the colour dielectric field,  $m_0$  is the u and d quark masses,  $m$  is the mass difference between s and u quarks,  $\sigma_v^2 = 2\alpha B/m_{GB}^2$ ,  $m_{GB}$  is the mass of the dielectric field and  $V(\chi)$  is the self interaction of the dielectric field;

$$V(\chi) = B(\alpha\chi^2 - 2(\alpha - 2)\chi^3 + (\alpha - 3)\chi^4).$$

In the mean field equations above, we have assumed a spherically symmetric and time independent dielectric field generated by the quarks present in the  $s_{1/2}$  orbital. (for the details of the CDM Lagrangian, field equations, mean field solutions etc the reader is referred to ref[21].) The wavefunction of a six-quark cluster is then constructed as a product of a six-quark space wavefunction and a spin-flavour-colour wavefunction. Since all the six quarks are in  $s_{1/2}$  orbital, the space wavefunction is symmetric. We find it convenient to express the spin-flavour-colour wavefunction as a product of two colour-neutral three-quark wavefunctions and then antisymmetrize the wavefunction with respect to the exchange of quarks between the two clusters:

$$|c_1 c_2 >_{f,s,c} = \mathcal{P} \sum_{1,2} \alpha_{1,2} |c_1 >_f |c_1 >_s |c_1 >_c \times |c_2 >_f |c_2 >_s |c_2 >_c \quad (3)$$

where the subscripts f, s, c denote the flavour, spin and colour wave functions of three-quark clusters and  $c_1$  and  $c_2$  denote the first and second cluster respectively. The colour wavefunction  $|c_i >_c = \sum_{l,m,n} \epsilon_{l,m,n} |l > |m > |n >$  is antisymmetric ( colour-singlet ) with respect to exchange and that is ensured by the Levi-Civita symbol  $\epsilon_{l,m,n}$ .  $\sum_{1,2}$  includes the summation over possible spins, isospins and hypercharges of the clusters 1 and 2 so as to give a dibaryon state of definite spin, parity, isospin and strangeness. The permutation operator

$$\mathcal{P} = \frac{1}{\sqrt{8}} [1 + \mathcal{S}_{14} \mathcal{A}_{14}^{fs}] [1 + \mathcal{S}_{25} \mathcal{A}_{25}^{fs}] [1 + \mathcal{S}_{36} \mathcal{A}_{36}^{fs}] \quad (4)$$

is required for proper antisymmetrization of quark wavefunctions between two clusters. Note that since the colour wavefunction of a cluster is a colour singlet, we

need to symmetrize intercluster colour wavefunction and therefore antisymmetrize the spin-flavour wavefunction. Also, the spin and flavour cluster wavefunctions are symmetric and therefore are simply the octate and decuplet baryon wavefunctions. The state thus constructed is a bare dibaryon state and corrections due to gluon and pseudoscalar meson interactions need to be included.

The dihyperon state we want to consider in this work is a colour- flavour- and spin-singlet state. In terms of the cluster wavefunction described above, the spin-flavour-colour wavefunction of the dihyperon is,

$$\begin{aligned}
|H_1 > = & \frac{1}{4} \mathcal{P} \{ |p\Xi^- > + |\Xi^- p > - |n\Xi^0 > - |\Xi^0 n > - |\Sigma^+ \Sigma^- > - |\Sigma^- \Sigma^+ > \\
& + |\Sigma^0 \Sigma^0 > + |\Lambda\Lambda > \} \{ \uparrow\downarrow - \downarrow\uparrow \} |C_1 >_c |C_2 >_c
\end{aligned} \tag{5}$$

where the first term on the right hand side of eq. (5) consists of a combination of baryon octet flavour wavefunctions and the second bracket is the antisymmetric (two baryon) spin wave function. Note that the baryon wavefunctions themselves consist of the product of SU(3) colour and SU(6) flavour-spin wave functions of quarks. The fact that the wavefunction  $|H_1 >$  is a singlet of colour and spin is obvious. One can convince oneself that it is a flavour singlet as well by showing that the operation of ( quark ) isospin, V-spin and U-spin raising and lowering operators on  $|H_1 >$  gives zero.

Let us now consider the mass of the dihyperon. In the cloudy bag model approach, the mass is given by

$$M_{H_1} = M_{bare} + M_c + M_{meson} \tag{6}$$

where  $M_{bare}$  is the contribution to the mass from the quarks and the dielectric field,  $M_c$  is the contribution due to colour magnetic energy and  $M_{meson}$  is the meson self-energy correction. The bare mass  $M_{bare}$  includes the energies of the quarks and the dielectric field. The energy due to the spurious center of mass motion is removed by using the Peierls-Yoccoz projection technique[26] in our calculation ( see ref[21] for

details). The colour magnetic energy  $M_c$  is given by

$$M_c = \frac{1}{2} \sum_{i < j} \int d^3r \vec{j}_i^a(\vec{r}) \cdot \vec{A}_j^a(\vec{r}) \quad (7)$$

where  $\vec{A}_j^a(\vec{r})$  is the ( colour ) vector potential generated by  $j^{th}$  quark and  $\vec{j}_i^a(\vec{r})$  is the colour current of  $i^{th}$  quark. The vector potential  $\vec{A}$  is obtained by using Green's function technique[21, 27]. Thus, we have

$$M_c = \frac{\alpha_s}{3} \sum_{i < j} \langle H_1 | \lambda_i \cdot \lambda_j \vec{\sigma}_i \cdot \vec{\sigma}_j \int r dr r' dr' \frac{g_i(r) f_i(r)}{\chi^4(r)} \frac{g_j(r') f_j(r')}{\chi^4(r')} G_1(r, r') | H_1 \rangle \quad (8)$$

where  $g$  and  $f$  are the upper and lower components of Dirac spinor,  $\chi^4$  is the colour dielectric function and the Green's function  $G_1(r, r')$  is defined in ref[21, 27].

The meson self energy  $M_{meson}$  is computed by coupling the pseudoscalar meson octate to the dihyperon. The meson coupling to the dihyperons leads to an octate of dibaryon states and the meson energy calculation requires the wavefunctions and masses of these states. The octate dibaryon wavefunctions are obtained by operating  $\sum_i \lambda_i^a \vec{\sigma}_i$  on the dihyperon state. Here  $\lambda_i^a$  are the ( flavour ) SU(3) Gell-Mann matrices for  $i^{th}$  quark and  $\vec{\sigma}_i$  is the spin operator. The masses of the dibaryon octate have been calculated by using the procedure outlined in the preceeding paragraphs.

The expression for the meson self energy is

$$\begin{aligned} M_{meson} = & \frac{3}{2f_\pi^2\pi^2} \left[ 3 \int \frac{k^4 dk v_\pi^2(k)}{\epsilon_\pi(k)(M_0 - M_\Sigma - \epsilon_\pi(k))} \right. \\ & + 2 \int \frac{k^4 dk v_K^2(k)}{\epsilon_K(k)(M_0 - M_N - \epsilon_K(k))} + 2 \int \frac{k^4 dk v_K^2(k)}{\epsilon_K(k)(M_0 - M_\Xi - \epsilon_K(k))} \\ & \left. \int \frac{k^4 dk v_\eta^2(k)}{\epsilon_\eta(k)(M_0 - M_\Lambda - \epsilon_\eta(k))} \right] \quad (9) \end{aligned}$$

Here  $M$ 's are the masses of the dibaryon states excluding the meson self energy, <sup>1</sup>  $v(k)$  is the form factor for the meson coupling to the quark in the dibaryon states and  $\epsilon(k)$  is the energy of the respective meson. Since the dihyperon state is a spin-flavour singlet,

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<sup>1</sup>The notation used for the dibaryon octate is same as that of the baryon octate. Thus,  $M_N$  is the mass of the dibaryon with isospin 1/2, hypercharge 1 and spin 1. It should not be confused with the nucleon mass



the pseudoscalar meson octate induces a transition between the dihyperon and the ( flavour ) octate of the dibaryon states having spin 1. The dibaryon octate states, in turn, couple to other dibaryon states through the coupling of the pseudoscalar meson octate. Thus, in a complete calculation, the coupling of the octate dibaryons with other dibaryon states should be included and the masses of all the dibaryon states should be determined in a consistent fashion. Such a somewhat formidable calculation is being done. Here we want to present the results of a restricted calculation described above.

We now come to the discussion of the results of our calculation. The parameters of the CCD model are the quark masses ( $m_0$  and  $m$ ), the strong coupling constant  $\alpha_s$ , the bag pressure  $B$ , the meson-quark coupling constant  $f$ , the mass of the dielectric field  $M_{GB}$  and the constant  $\alpha$ . Of these, we have fixed  $f = 93MeV$  ( the pion decay constant ) and  $\alpha = 24$ . Rest of the parameters have been chosen by fitting the octate and decuplet baryon masses. Earlier[21] we have shown that the fitting procedure yields a limited range of values of  $m_0$ ,  $B$  and  $m$  ( $m_0 \sim 100MeV$ ,  $m \sim 200MeV$ , and  $100MeV < B^{1/4} < 140MeV$ ) where as the strong coupling constant  $\alpha_s$  is essentially determined by nucleon-delta mass difference. But equally good fits to the baryon masses are obtained for a wide range of the glueball mass ( $0.8GeV < m_{GB} < 1.5GeV$ ). However, it has been found that the lower values of glueball mass yield better values of charge radii and magnetic moments. In the present calculation the dihyperon mass has been computed for a large set of the parameter values which fit the masses of nucleon,  $\Delta$  and  $\Lambda$  masses. For these sets, the difference between calculated masses of other octate and decuplet baryons and the experimental masses is within few %. Moreover, the variation in the individual baryon masses over the range of the parameter set considered here is only 10-15 MeV.

The calculated dihyperon masses are displayed in Table 1. In the results shown here the glueball mass has been varied from 800 MeV to 1250 MeV, the u and d quark mass is varied between 100 and 135 MeV and  $B^{1/4}$  is varied between 100 and 125 MeV. It is interesting to note that the dihyperon mass increases almost linearly with the

Table 1: The dependence of dihyperon mass ( $M_0$ ) on the parameters of the CCD model.  $\alpha_s$  is dimensionless and the masses are in the units units of MeV.

$m_{GB}$	$\alpha_s$	$m_0$	$m$	$b^{1/4}$	$M_0$
819.	.269	103.	211.	103.	2071.
893.	.474	123.	210.	104.	2073.
928.	.288	108.	212.	108.	2087.
967.	.581	133.	209.	106.	2083.
1008.	.216	102.	213.	113.	2103.
1059.	.272	111.	213.	115.	2109.
1118.	.260	112.	213.	118.	2119.
1168.	.436	132.	211.	118.	2121.
1207.	.232	112.	214.	123.	2133.
1251.	.215	111.	215.	125.	2140.

glueball mass and does not show any systematic dependence on the other parameters. Further more, the variation in the dihyperon mass is quite large. For example, the dihyperon mass changes by about 70 MeV when the glueball mass is increased from about 800 MeV to 1250 MeV. This variation in the dibaryon mass arising from the change in  $m_{GB}$  is about an order of magnitude larger than the variation found in the baryon octate and decuplet masses. Here we would like to note that the lower values of the glueball masses (  $m_{GB} < 1GeV$  ) yield better agreement with the static properties of baryons ( charge radii, magnetic moments etc ) [21]. Further more, it has been observed that a better agreement with the  $\pi N$  scattering data is obtained for  $m_{GB} \sim 1GeV$  or smaller. We therefore feel that results with  $m_{GB} \leq 1GeV$  are somewhat more realistic.

The results in Table 1 show that the computed dihyperon masses are smaller than the  $\Lambda - \Lambda$  threshold. Thus, the dihyperon is stable against strong decays in the CCD model. The binding energy of the dihyperon varies between  $160MeV$  ( for  $m_{GB}$  of  $800MeV$  ) and  $90MeV$  ( for  $m_{GB}$  of  $1250MeV$  ). These values are larger than the result of S. Ahmed et al. [20] as well as Jaffe's prediction [1].

To conclude, we have calculated the dihyperon mass using CCD model. Along with the colour magnetic energy, we have also investigated the effect of the quark-meson coupling on the dihyperon mass. The correction due to the spurious motion of the center of mass is included in the calculation. The projection technique is used to project out the good momentum states and these states are used in the computation of the dibaryon-meson form factors. It is found that the dihyperon is stable against the strong decays for the parameters of the CCD model considered in the calculations with the binding energy of about 100 MeV or more. The determination of the dihyperon width ( due to the weak interaction ), masses of other dibaryons and their strong decay widths ( due to their decay into a pair of baryons ) in the CCD model needs to be done. These calculations are in progress.

Our results are significant in the context of ongoing search for the quark-gluon plasma in the laboratory. One of the possible unambiguous way to detect the transient

existence of a temporarily created QGP might be the experimental observation of exotic remnants like the formation of strange matter or strangelets [29, 30]. The six-quark dihyperon state is supposed to be the lightest strangelet state. So the fact that such states are found to be stable for the reasonable parameter ranges in the present study, makes it imperative to put more experimental efforts to detect such objects.

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